

Development of Creativity Using 3D Dynamic Geometry System GInMA

Vladimir Shelomovskiy
e-mail: vvsss@rambler.ru
Murmansk State University
Russia

Svetlana Nosulya
e-mail: svedeni@yandex.ru
Deoma
Russia

Abstract

Experience of teaching mathematics with dynamic geometry system GInMA is described in this article. GInMA is the part of Interactive Mathematical Art (InMA) project, which has been used for teaching mathematics in Russian schools with in-depth study of mathematics since 2005. InMA is a tool for creating electronic textbooks; courses in algebra and geometry have been developed.

InMA project includes computer algebra system (CAS), single-stepping system (SSS), graphing tools for all kinds of functions and two kinds of dynamic geometry systems (DGS). That allows creating interactive 2D and 3D geometric images, interactive graphics and text with changeable parameters. All the constructions in the geometric part of InMA project are carried out on the screen. However, teachers prefer to use ready-made teaching packages on the topics, only adding some more lines if it is need to answer questions of curious students.

Many complex geometric problems can be represented as logical chains and a process of constructing of such a logical chain helps to develop student's creativity. DGS help to build logic chain, allowing step-by-step to provide a common part of overlapping solids. In this paper we consider the samples of methodical sets created using dynamic geometry system GInMA.

All GInMA files that accompany this paper can be found in GInMA library.

Install the GInMA software from the website

<http://deoma-cmd.ru/en/Products/Geometry/GInMA.aspx>

Open GInMA library, choose "Papers" book in a books selection dialog, open "eJMT Development of creativity using 3D DGS GInMA" or "eJMT Yoichi Maeda papers illustrations", open the GInMA files and investigate interactive solutions of problems.

1. Introduction

Experience of teaching mathematics with Interactive Mathematical Art (InMA) software is described in this article. InMA project is used for teaching mathematics in Russian schools with in-depth study of mathematics since 2005. Courses in algebra and geometry have been developed for secondary school based on the traditional textbooks of several popular Russian authors. Now about 1500 students and 60 teachers use InMA software, 20 teachers act as experts, that is periodically make suggestions to improve on topics. The teachers have a special interest in interdisciplinary methodical sets, examples are demonstrated in this article.

The project developer's team was guided by the idea that the use of computer tools for educational purposes requires an integrated approach. This should be a sequence of steps: concept creation, computer program development, electronic textbook creation with the use of this program and printed textbook data base, use of the electronic textbook by school teachers, the joint modification of the software by programmers, mathematicians and teachers. The underlying concept is the desire to maximize the interactive features of the computer in teaching, to make creative teaching, which provides maximum visualization of algebraic concepts, ample opportunity to change the settings, vivid geometric constructions. This product should be convenient for the

teacher, both from a pedagogical point of view and in terms of application in the classroom during the lesson and at home schooling.

Interactive Mathematical Art (InMA) project includes computer algebra system (CAS), single-stepping system (SSS), graphing tools for all kinds of functions and two kinds of dynamic geometry systems (DGS). Programs are written in languages D (using Gtk) and C # (Net Framework, Windows Forms). InMA programs are now collected under OS Windows, but the transfer to Linux and Mac is not complicated.

InMA allows creating interactive 2D and 3D geometric images, interactive graphics and text with changeable parameters. Interactive image is close to that used by teachers on the blackboard in the educational process. The formulae are developed in such a way that the brackets, roots, denominators appear and disappear automatically when interactive factors are changed. Single-stepping system interactive formulae ensure the conclusion of the chains of conversions in the customary form, the figures in the customary notation. Program ensures the functioning of interactive points and graphics (limitation of motion by curve, by surface; rational coordinates).

InMA project is a tool for creating electronic textbooks and methodical sets. The electronic textbook idea lies in the fact that the text of traditional textbook becomes the basis, on which tasks of the same kind are created. A teacher gets a kind of a semi-finished textbook with the opportunity of selecting a necessary variant from several others. Each file is provided with a text on methodology of teaching. The basic formulae, tasks and control elements of the file, recommended installations and terminology are indicated there. Algebraic part of the project is designed so that the teacher has no way to change the formulae, but can modify a wide range of parameters of these formulae. Initial formulae are programmed. All the constructions of the geometric part are carried out on the screen. However, the teachers prefer to use ready-made teaching packages on the topic, only adding some more lines if it is needed to answer questions of curious students.

Demo versions of InMA Project can be accessed at <http://deoma-cmd.ru/>

In this paper we consider samples of methodical sets created using dynamic geometry system GInMA, the geometrical part of Interactive Mathematical Art (InMA) project. All GInMA files that accompany this paper can be found in GInMA library. Install the GInMA software from the website <http://deoma-cmd.ru/en/Products/Geometry/GInMA.aspx>

Open GInMA library, choose "Papers" book in a books selection dialog, open "eJMT Development of creativity using 3D DGS GInMA" or "eJMT Yoichi Maeda papers illustrations" and open the GInMA files for the investigation of interactive solutions of problems.

2. Construction of Logical Chains

There are many complex geometric problems, the solution of which requires the integration of imagination and logic. The use of them in the form of logical chains develops students creativity. Dynamic geometry system GInMA helps to build logic chain, allowing a step-by-step method to provide a common part of overlapping solids and deal with it in different ways.

2.1 Logical Chain with $(n - 1)$ Dimensional Objects that Divide the n -dimensional Space

Consider the possible construction of a logical chain, a useful method for studying the mathematical induction and connections between seemingly unrelated phenomena. The task is to find the number of parts into which the objects of dimension $n - 1$ divide the n -dimensional space. The initial problems are: to find the number of parts N into which n points divide a line or a circle. The results are simple, for example $N(n + 1) = N(n) + 1$. GInMA file contains the interactive solution of the problem concerning the curve divided by points [1]. The next obvious level is to

find the number M of parts into which n lines (or circles) of the total or nearly total divide a plane or a sphere. At this step $M(n + 1) = M(n) + N(n)$. Here we have easy to understand two-dimensional drawings. GInMA file contains the interactive solution of the problem concerning a plane divided by the circles [2]. And it is not clear why do we need to use mathematical induction. But at the next step the calculation in space makes clear need for induction and dynamic geometry system, which allows us to disassemble the intersecting objects, to keep the lines of intersection of the solids, to look at the objects from different angles, to look inside the objects. Even such a simple calculation, as the search for the number of intersection points of four spheres raises the need for finding the methods of calculation (e.g. double the calculation), because the imagination is clearly not enough. And at the same time the results are obvious and have attracted considerable interest of students. Here the method of transferring information from the previous level is used, where the objects are easily manageable. The last level, where dynamic geometry system is particularly useful, is to find the number of parts into which n planes (or spheres) of the total or almost a generic divide the space. It is useful to examine the symmetry of shapes, studying the equal spheres with centers lying at the vertices of Platonic solids. From the intersection of solids we should pass to the traces on one of them. GInMA files contain interactive solutions [3],[4].

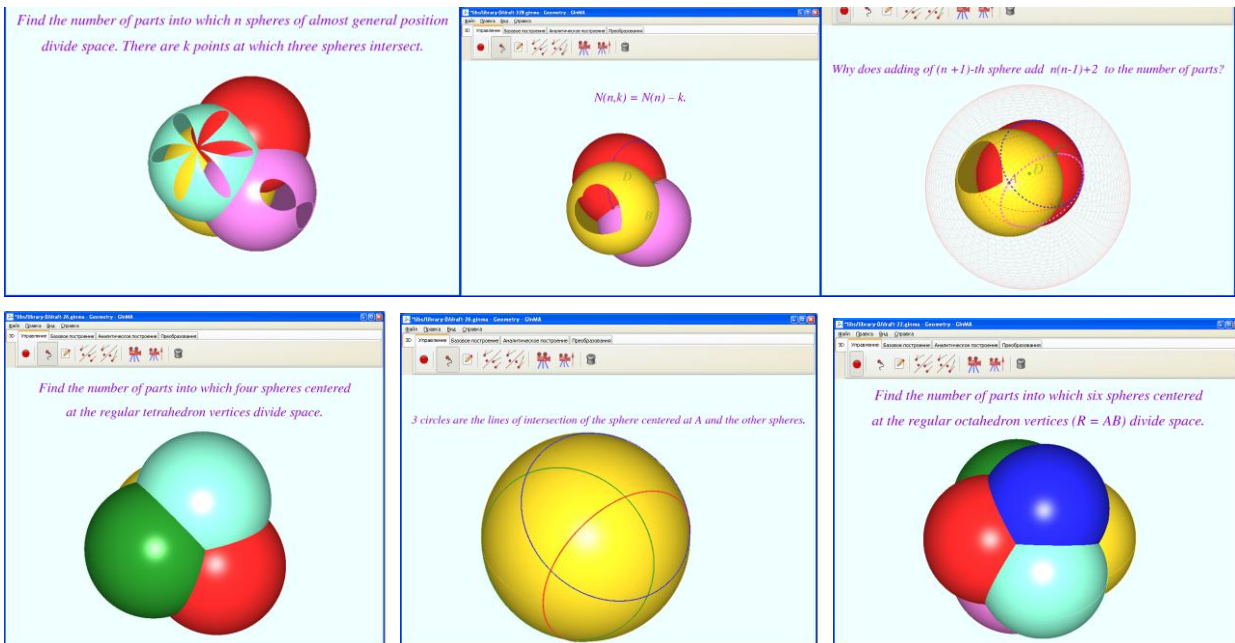


Figure 2.1.1 Division of Space by Intersecting Spheres. (Plots produced with [GInMA], [3].)

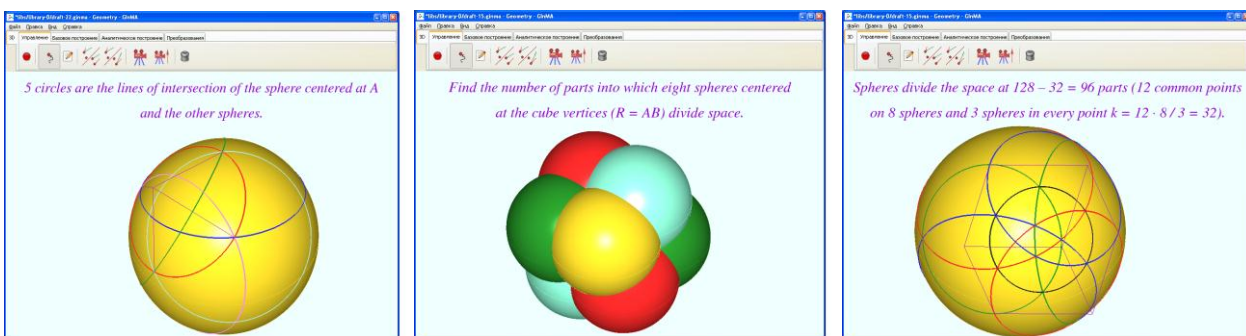


Figure 2.1.2 Division of Space by Intersecting Spheres. (Plots produced with [GInMA], [4].)

2.2 Logical Chain Based on the Cavalieri's Principle

Logical chain based on the Cavalieri's principle (see [Cavalieri's principle]) may help to hone a skill at calculating double and triple integrals. The initial problem is to find the volume of a sphere. Relatively simple tasks are: to find the volume of a spherical segment, a watermelon wedge, a ring and a hoof (the part of a cylinder that is cut off from the cylinder by a plane). Further, the 3-D shapes obtained as a result of rotation of a plane region about an axis of rotational symmetry are being investigated. GInMA files contain the interactive solutions of the problems concerning the finding of volume of a spiral solid [5], a solid of revolution formed by rotating a semicircle [6], a solid of revolution formed by rotating a cube around the diagonal [7]. We can replace the solid of revolution and change the axis of rotational symmetry. Relationship with physics is effectively demonstrated by the problems concerning the volume of a solid of revolution, as the distance between the center of mass and the axis of rotation is given in the formula. Further we study the rotation of 3-D shapes, for example, rotation of the cube about the diagonal and about the edge diagonal. The next step is to find the the configuration of the solids that are a common part of two identical cylinders intersecting at right and at arbitrary angles (GInMA file contains the interactive solution of the problem [8], video shows the solution of the problem of solid common to two intersecting cylinders using GInMA [25]); a common part of three or more identical cylinders, which intersect in various ways. Further, the solids arising from the intersection of a sphere and a cylinder are studied (GInMA file with the interactive solution [9]). At this step we can consider different variations of the problem. For example, we may take the problem of Viviani's curve (see [Viviani's curve]), connecting points on the Earth's surface at the same latitude or longitude. Samples are in the Figures 2.2.1 – 2.2.4, GInMA interactive solutions are [5],[6],[7],[8],[9].

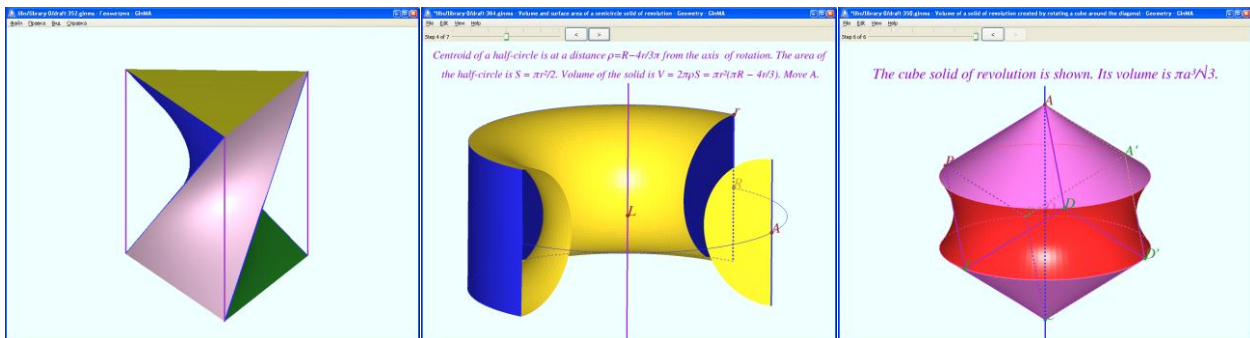


Figure 2.2.1 Problems Involving Solids of Revolution (Plots produced with [GInMA],[5], [6], [7].)

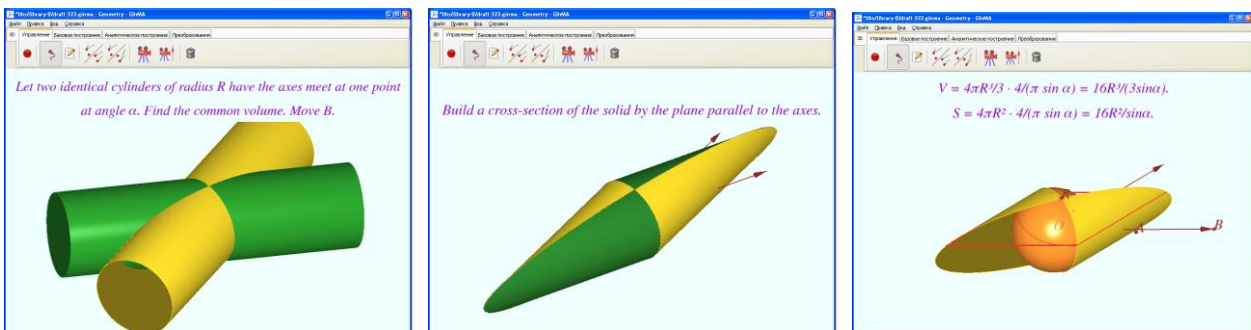


Figure 2.2.2 Solid Common to Two Intersecting Cylinders.(Plots produced with [GInMA], [8].)

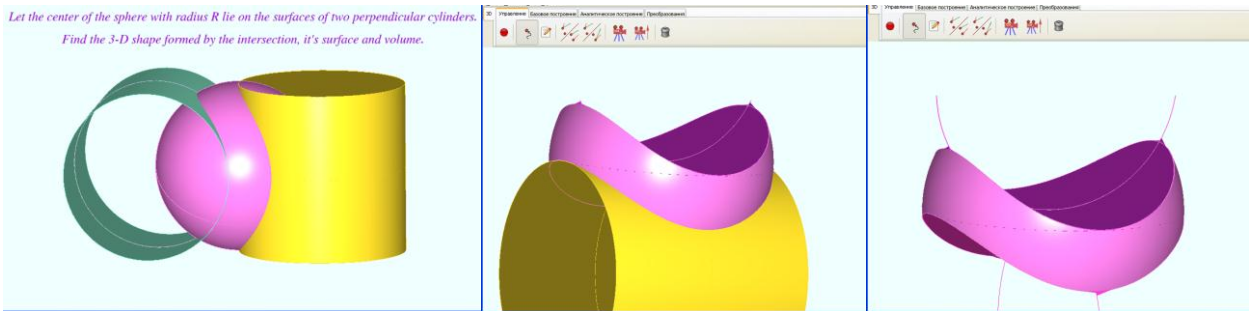


Figure 2.2.3 Two Perpendicular Cylinders Intersect a Sphere. (Plots produced with [GInMA], [9].)

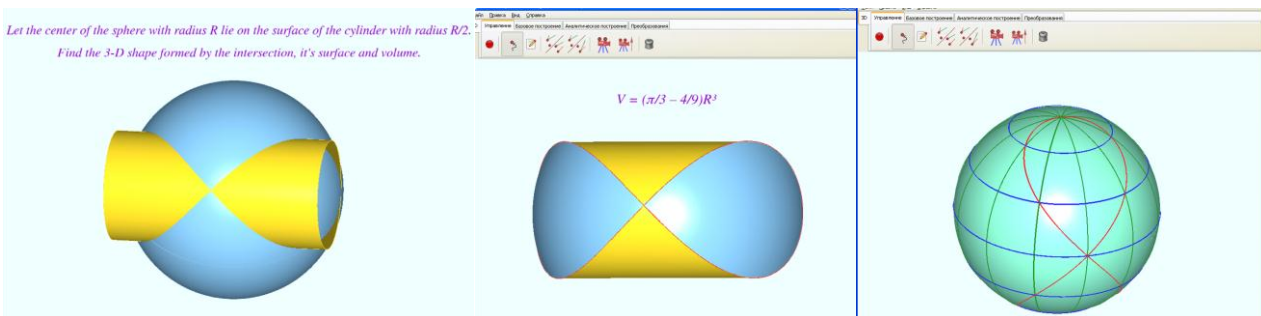


Figure 2.2.4 3-D Shape Formed by Intersection of a Sphere and a Cylinder. (Plots produced with [GInMA])

2.3 Logical Chain for the Method of Mirrors

Consider the possible construction of a logical chain for the method of mirrors. The basic task is to build the shortest path between two points, passing through the line if the line and the points lie in one plane. The development of this problem is to find the shortest path within a triangle (a rectangle, a diamond), which contains points on each side. An important feature, to which attention should be paid, is that the triangle has a unique way, and the rectangle has many ways of equal length, and the center of the rectangle is the center of symmetry of each path. The next step is to find a similar path in a regular tetrahedron, an arbitrary tetrahedron, a cube, a cuboid and a regular octahedron. Constructing similar paths, we get plenty of opportunities to discuss: one path is in the tetrahedron, the set is in the cube; the plane of the pairs of adjacent parts of the trajectory is perpendicular to the facet plane (property of the reflected beam); the trajectory has central symmetry, etc. The example is in the Figure below, GInMA interactive solution is [10].

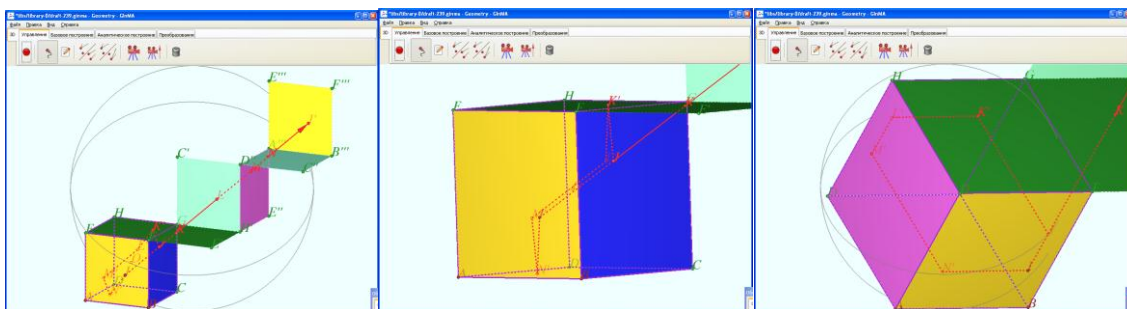


Figure 2.3.1 Construction of a Shortest Closed Path inside a Cube. (Plots produced with [GInMA], [10].)

2.4 Logical Chain Based on Finding the Volume of Polyhedral Intersection

The logical chain based on finding the volume of polyhedral intersection may begin from the intersection of regular tetrahedrons, one obtained from another by rotation about an axis passing through the midpoints of opposite edges. This task allows you to make an elementary calculation of the total volume for an arbitrary rotation angle. The problem concerning the volume of the solid in the case of a cube cut by dihedral angle, when the angle edge lies on the cube diagonal, is reduced to finding equal-area cross-sections of the cube being cut by half-planes. If the angle between the half-planes is 60° , then the largest and the smallest volumes are the same.

Visualization is useful for understanding the Newton's extremality principle and allows comparing the features of the integration method and the elementary computation methods.

The solving of such problems leads to the problem of the analysis of the accuracy of the standard mathematical software. For example, the problem is: we must build a solid of intersection of two cubes, one vertex of which lies in the center of another [11]. By Maple 12 the corresponding equations can be written, they are solved and the result can be compared with the geometrically exact solution. We were surprised to find the notable difference of these solutions: one and a half per cent. We'll be glad if someone will verify our solution.

The samples are shown in the Figures 2.4.1, 2.4.2, GInMA file with interactive solving is [11].

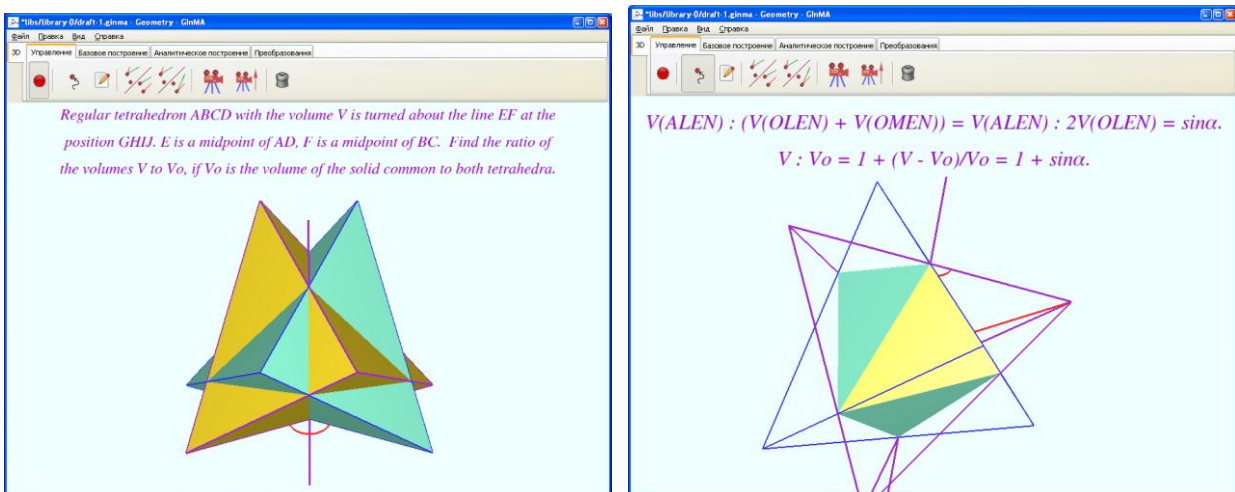


Figure 2.4.1 Problem Concerning the Intersection of Regular Tetrahedrons. (Plots produced with [GInMA].)

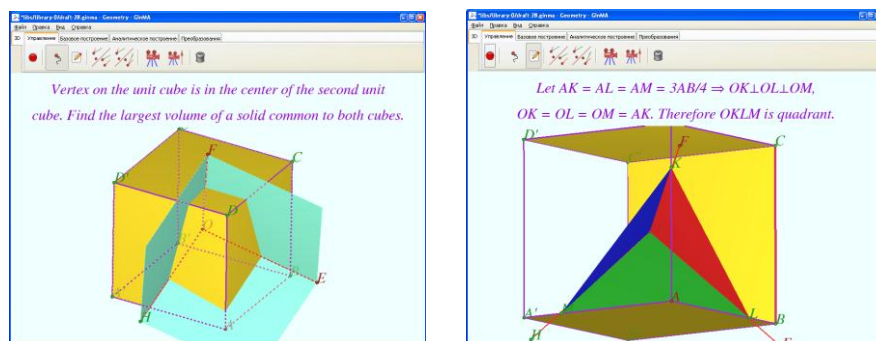


Figure 2.4.2 Problem Concerning the Intersection of the Cubes. (Plots produced with [GInMA], [11].)

3. Development of Geometric Imagination

Dynamic geometry system gives a great opportunity for research. For example, let us consider the problem involving the location of three regular tetrahedrons inside a cube when the tetrahedron edge is equal to the cube edge [12]. This problem allows the teacher to find the student's difficulties in space image understanding. It becomes obvious by studying the actions that students take in order to establish the tetrahedron in the required position relative to the cube. Another example is the problem concerning the number of circles with given radius at given center that can be accommodated touching the faces of the quadrant [13]. The samples are shown in the Figures 3.1, 3.2, GInMA files with interactive solving are [12], [13].

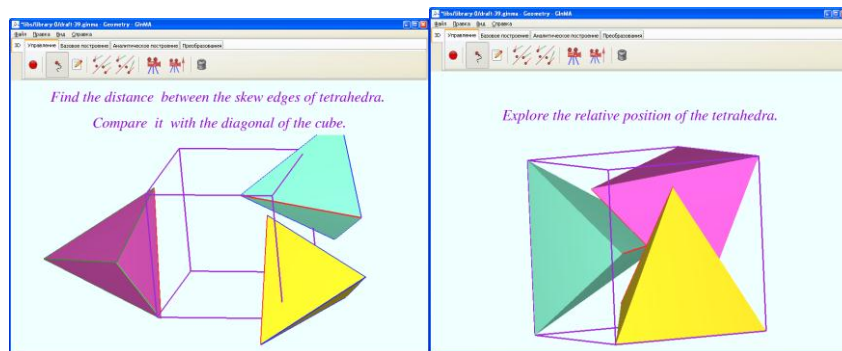


Figure 3.1 Three Regular Tetrahedra Inside a Cube. (Plots produced with [GInMA], [12].)

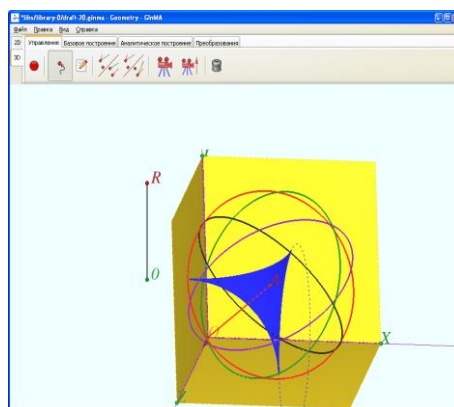


Figure 3.2 Circle Tangent to All Faces of Quadrant. (Plot produced with [GInMA], [13].)

4. Visualization of Complex Geometric problems

The more complicated problems students have to solve, the more developed their way of thinking becomes, the wider range of methods and techniques they use in problem solving. At the same time much too complicated problems dealing with abstract concepts at which only the most gifted students are skilled can be used for teaching many students, if we apply the visualization of complex mathematical constructions. Let us consider the opportunities of such visualization by creating illustrations using dynamic geometry system GInMA to problems from Yoichi Maeda papers [Maeda, 2008], [Maeda, 2010], [Maeda, 2011].

4.1 Angle on a Sphere

Let us consider the problem, the solution of which is a part of main problem in Yoichi Maeda paper [Maeda, 2008].

Problem: Let a triangle ABC lie on the unit sphere S^2 centered at O . Let AOB be a fixed angle determined by three points O, A , and B in the three dimensional Euclidean space. We want to find the locus of points of the sphere from which the arc is visible at constant angle. In another word, dihedral angle between the planes AOC and BOC is fixed.

In [Maeda, 2008] Section 2 named “Contour of visual angle on S^2 ” Figures 2.2 and 2.3 (made in Maple) are shown. GInMA illustrations you can see in GInMA library as “Locus of points of the sphere” [14]. The screenshots are shown on Figures 4.1.1– 4.1.4.

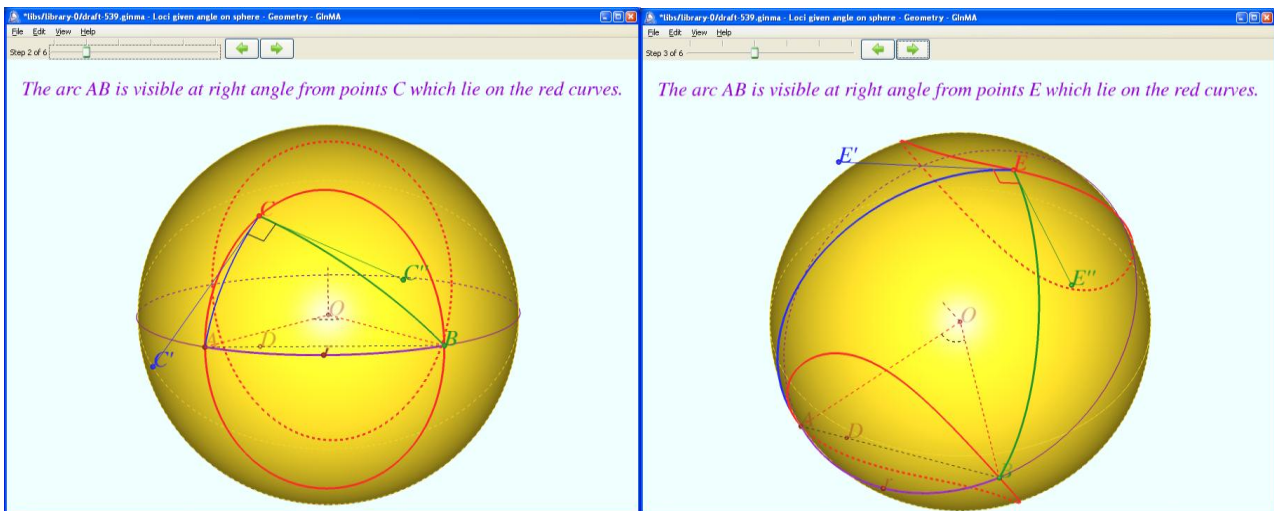


Figure 4.1.1 The curves from any point of which the arc AB is visible at right angle.(Plots produced with [GInMA], [14].)

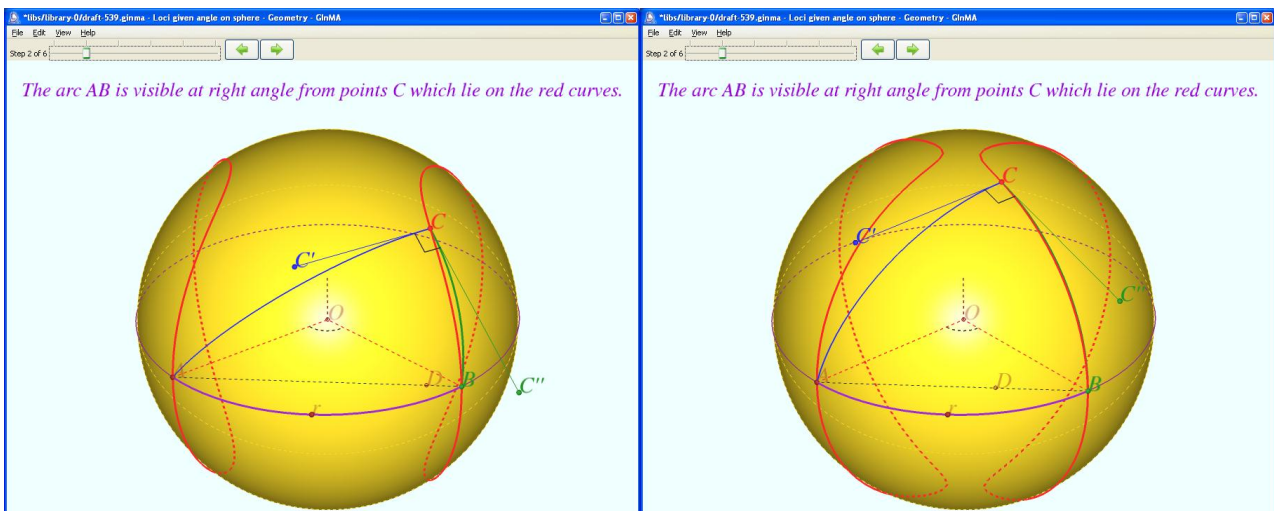


Figure 4.1.2 The curves from any point of which the arc AB is visible at right angle.(Plots produced with [GInMA], [14].)

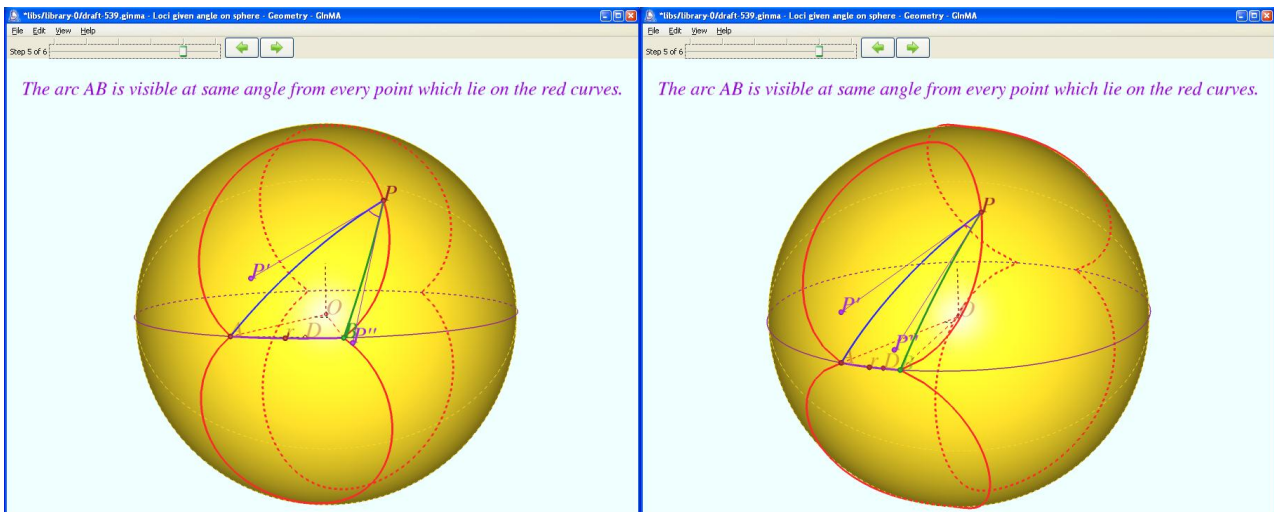


Figure 4.1.3 The curves from any point of which the arc AB is visible at an acute angle equal to APB . (Plots produced with [GInMA], [14].)

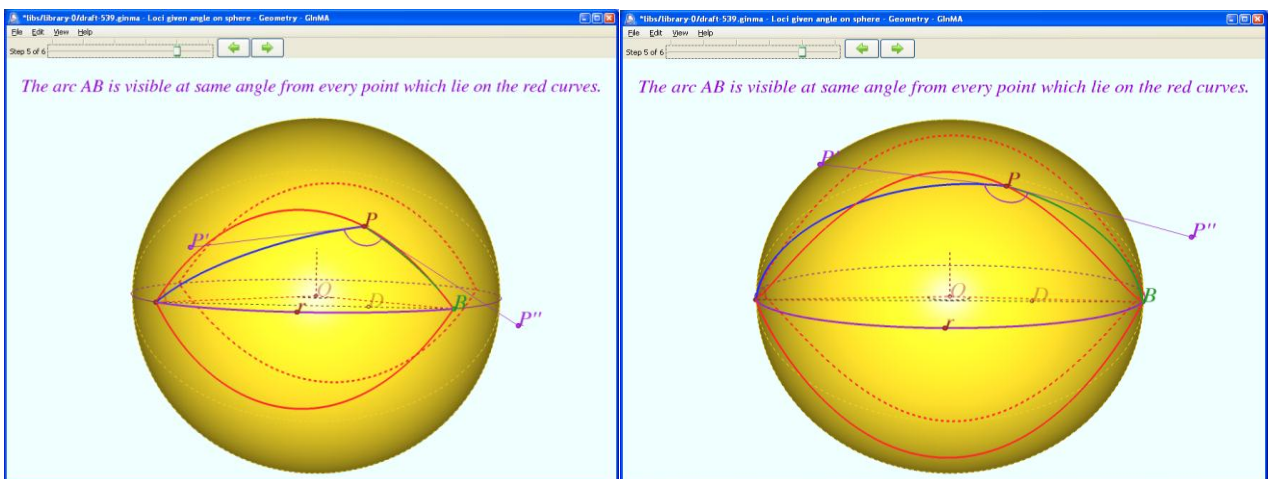


Figure 4.1.4 The curves from any point of which the arc AB is visible at an obtuse angle equal to APB . (Plots produced with [GInMA], [14].)

4.2 Common Perpendicular to the Geodesics in Three-dimensional Hyperbolic Space H^3

Let us consider another problem from Yoichi Maeda paper [Maeda, 2010].

Problem: Geodesic (non-Euclidian) lines AB and CD are given in three-dimensional hyperbolic space H^3 . Construct a common perpendicular to these geodesics.

In [Maeda, 2010] the author makes step-by-step solution of the problem. GInMA illustrations for every step have been created to make easier understanding of solution methods, which deal both with 2D and 3D spaces [15] - [22].

There is video showing the solution of this problem using GInMA [26].

Let us consider solution to the problem with the screenshots of GInMA files, Figures 4.2.1-4.2.9.

In [Maeda, 2010] Figure 2.1 “Geodesic line in the Poincare model” is shown. Corresponding file in GInMA library is “Line in two-dimensional hyperbolic space H^2 ” [15].

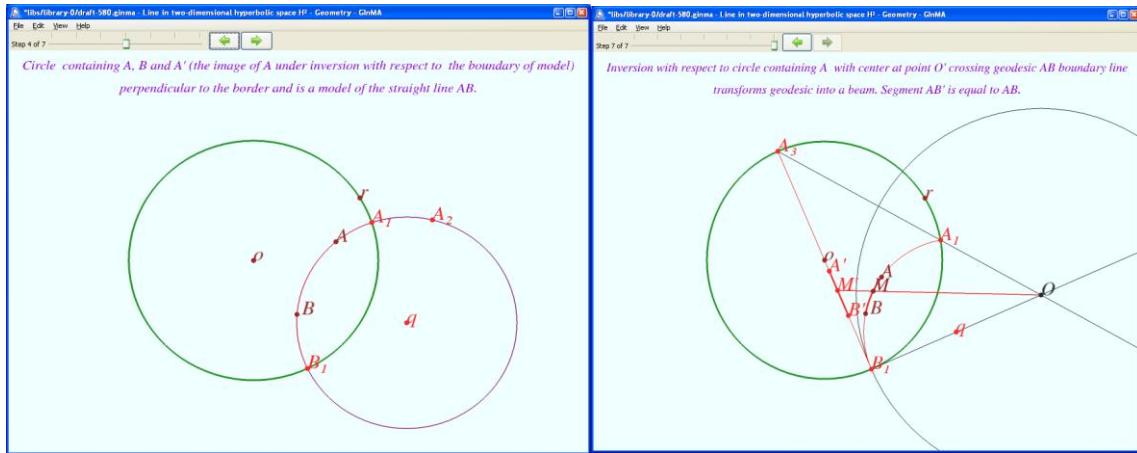


Figure 4.2.1 Line in two-dimensional hyperbolic space H^2 . (Plots produced with [GInMA], [15].)

In [Maeda, 2010] Figure 2.2 “Perpendicular line in the Poincare model” is shown. Corresponding file in GInMA library is “Perpendicular to a line in the H^2 model” [16].

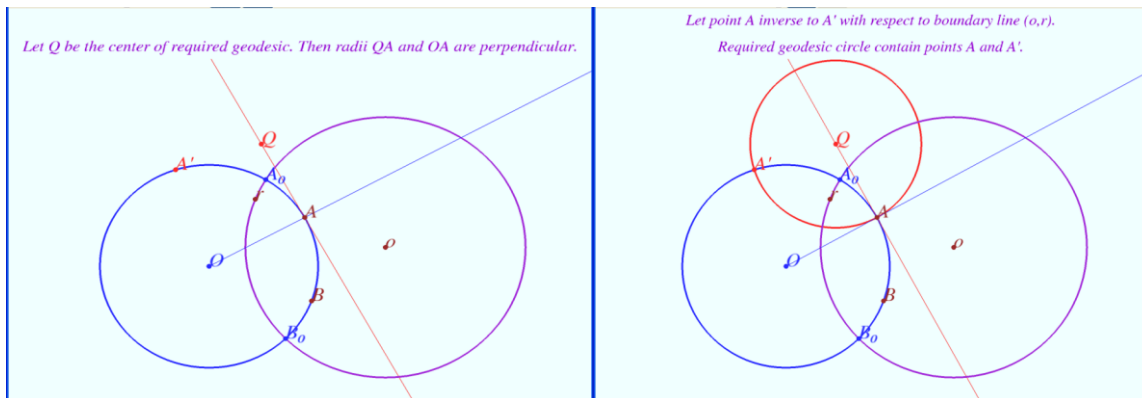


Figure 4.2.2 Perpendicular to a line in the H^2 model. (Plots produced with [GInMA][16].)

In [Maeda, 2010] Figure 2.3 “Perpendicular line in the half-plane model” is shown. Corresponding file in GInMA library is “Perpendicular to line in the half-plane model” [17].

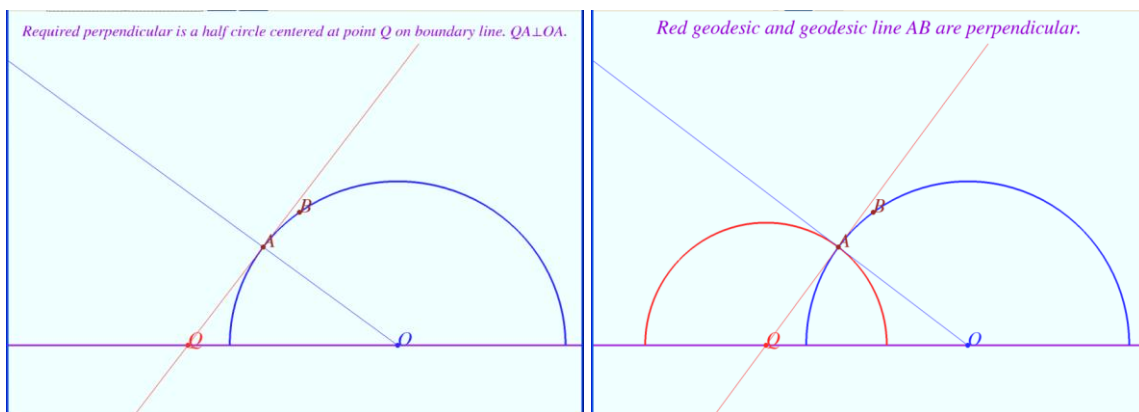


Figure 4.2.3 Perpendicular to line in the half-plane model. (Plots produced with [GInMA][17].)

In [Maeda, 2010] Figure 2.4 “Perpendicular line in the half-space model” is shown.

Corresponding file in GInMA library is “Perpendicular line in the half-space model” [18].

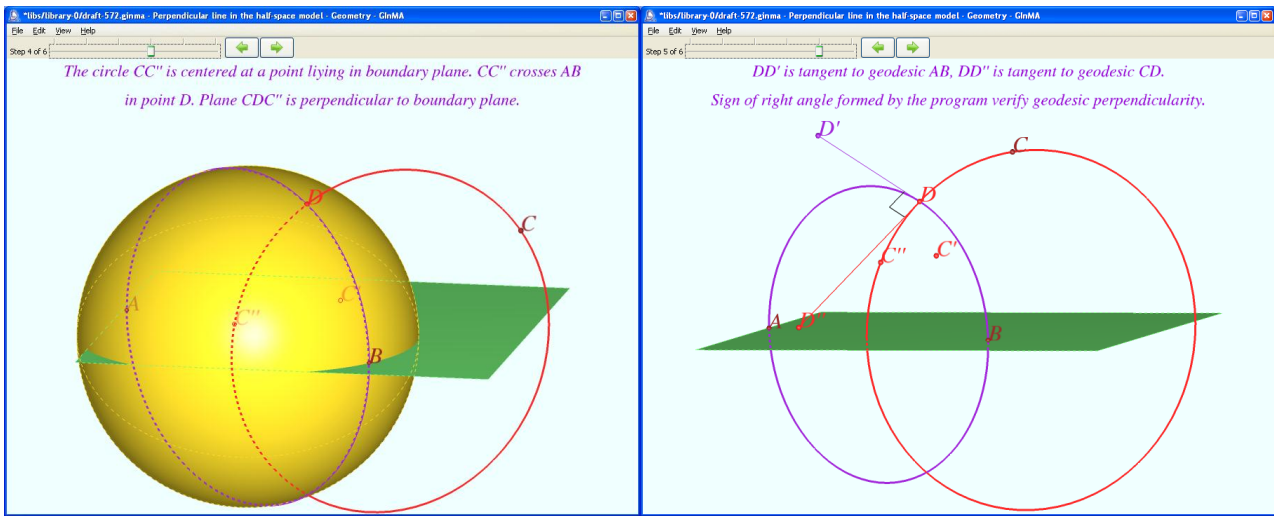


Figure 4.2.4 Perpendicular line in the half-space model. (Plots produced with [GInMA][18].)

In [Maeda, 2010] Figure 3.1 “Inverse image of the stereographic projection (left) and its top view (right)” is shown.

Corresponding file in GInMA library is “Perpendicular circles on the sphere” [19].

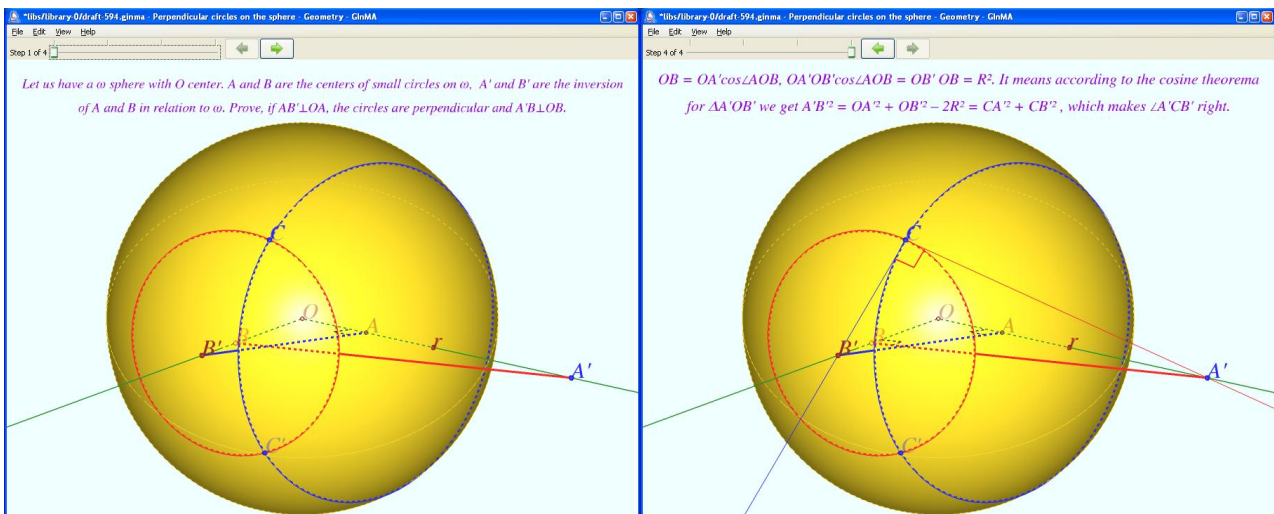


Figure 4.2.5 Perpendicular circles on the sphere. (Plots produced with [GInMA][19].)

In [Maeda, 2010] Figure 3.2 “Common perpendicular in the half-space model (special case)” is shown. Corresponding file in GInMA library is “Common perpendicular in the half-space model (special case)” [20].

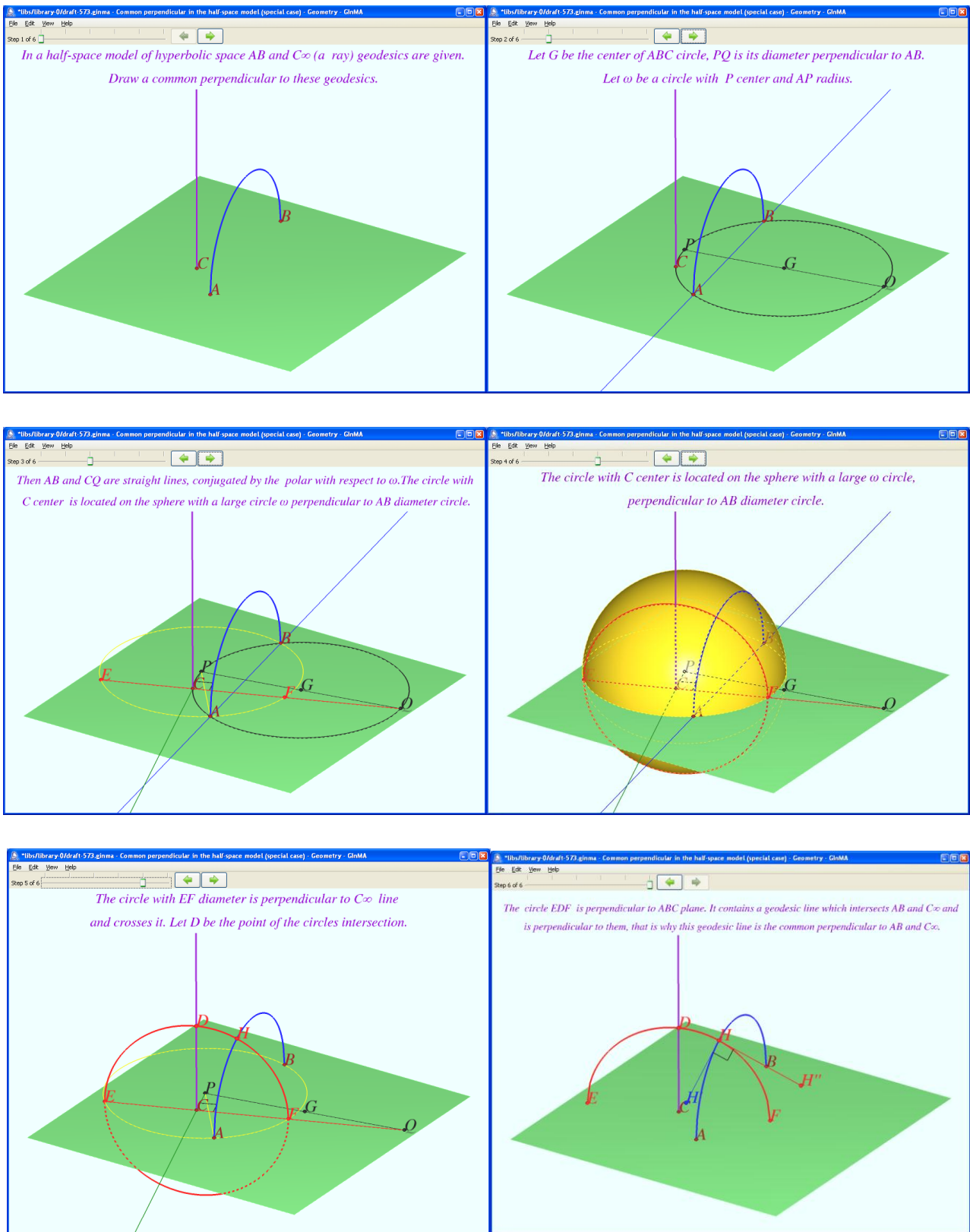


Figure 4.2.6 Common perpendicular in the half-space model (special case).
(Plots produced with [GInMA][20].)

In [Maeda, 2010] Figure 3.3 “Common perpendicular in the half-space model (general case)” is shown.

Corresponding file in GInMA library is “Common perpendicular in the half-space model” [21].

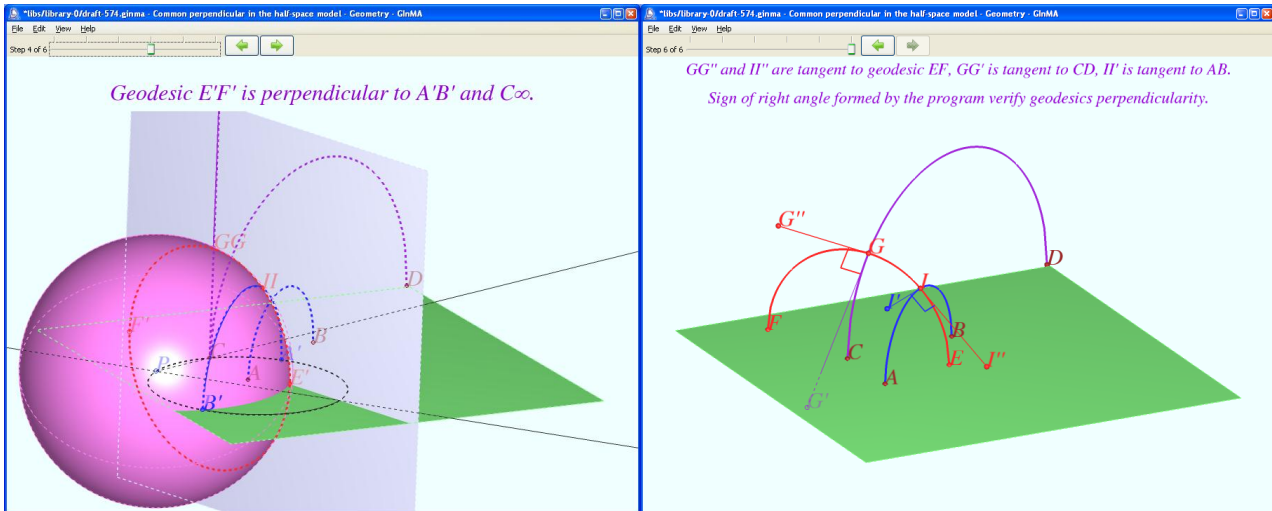


Figure 4.2.7 Common perpendicular in the half-space model. (Plots produced with [GInMA],[21].)

In [Maeda, 2010] Figure 3.4 “Common perpendicular in the Poincare model” is shown.

Corresponding file in GInMA library is “Common perpendicular in the H^3 Poincare model” [22].

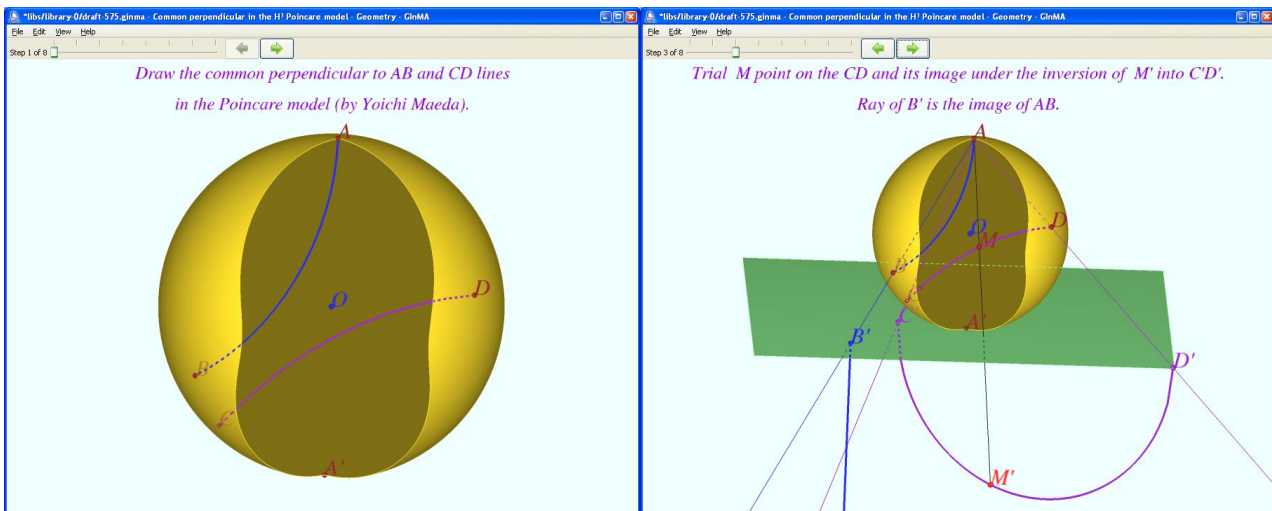


Figure 4.2.8 Common perpendicular in the H^3 Poincare model (1)

(Plots produced with [GInMA], [22].)

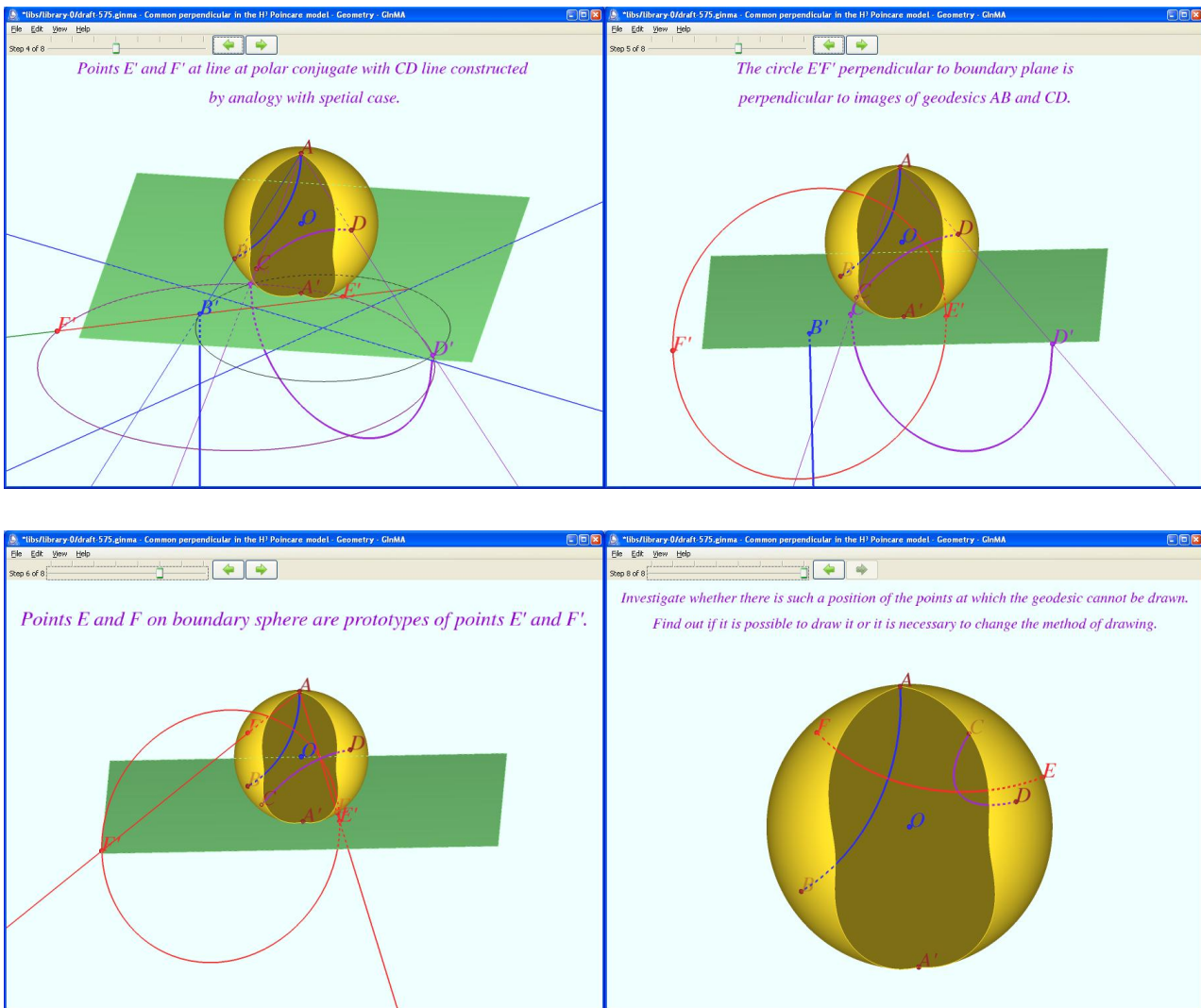


Figure 4.2.9 Common perpendicular in the H^3 Poincaré model (2)(Plots produced with [GInMA])

4.3. Construction of the Common Perpendiculars in the Three-sphere.

Let us consider the problem from Yoichi Maeda paper [Maeda, 2011]. In this paper the author solve the problem of construction of the common perpendiculars in the three-sphere. The part of this problem is special case: Section 4. Construction of the common perpendicular in S^3 (p.157, [Maeda, 2011]).

Problem. Let given circle g_1 and Euclidean line g_2 be in common position. Construct a circle perpendicular to g_1 with a diameter lying on g_2 .

[Maeda, 2011] contains “Construction 3. (Common perpendicular in the special case)”. Here the author makes in detail construction procedure and we use steps numeration from it in our GInMA illustrations. You can see our file in GInMA library as “Common perpendicular to the line and the circle” [23].

Let us consider solution to the problem with the GInMA file [23] screenshots, Figures 4.3.1-4.3.10.

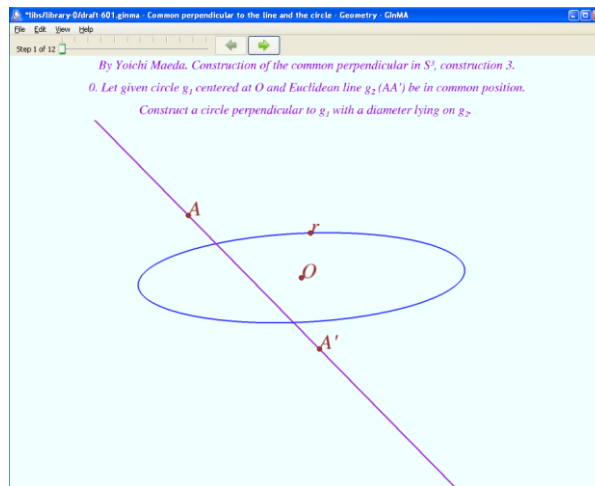


Figure 4.3.1 Step 0. Let given circle g_1 centered at O and Euclidean line g_2 (AA') be in common position. Construct a circle perpendicular to g_1 with a diameter lying on g_2 .

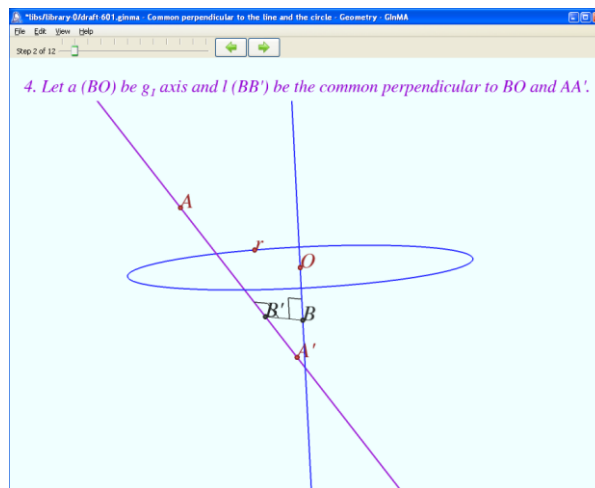


Figure 4.3.2 Step 4. Let a (BO) be g_1 axis and l (BB') be the common perpendicular to BO and AA' .

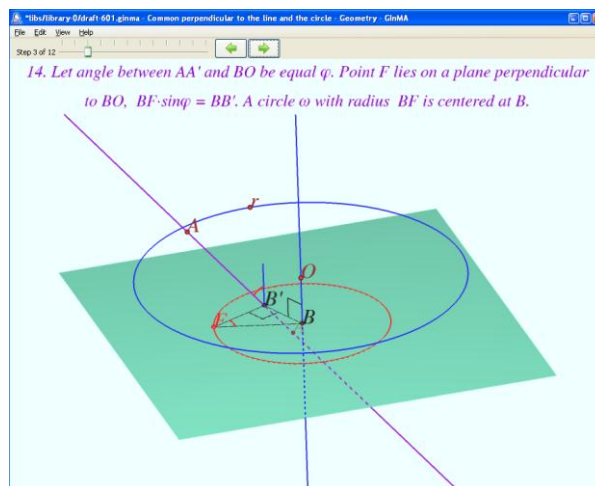


Figure 4.3.3 Step 14. Let angle between AA' and BO be equal φ . Point F lies on a plane perpendicular to BO , $BF \cdot \sin\varphi = BB'$. A circle ω with radius BF is centered at B .

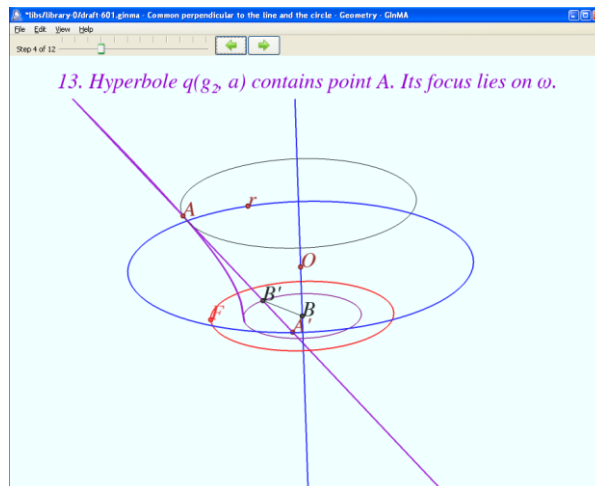


Figure 4.3.4 Step 13. Hyperbole $q(g_2, a)$ contains point A . Its focus lies on ω .

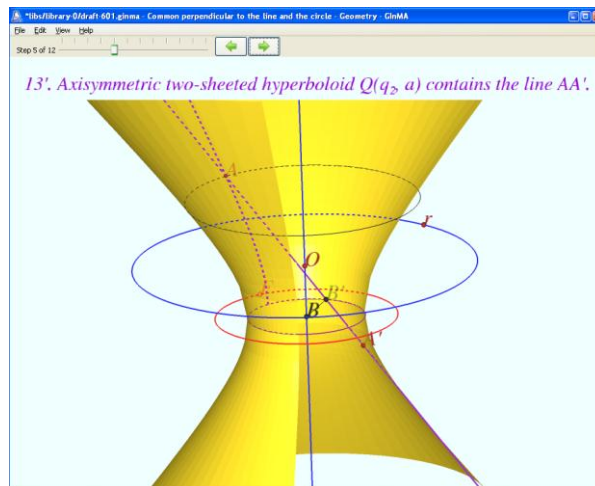


Figure 4.3.5 Step 13'. Axisymmetric two-sheeted hyperboloid $Q(g_2, a)$ contains the line AA' .

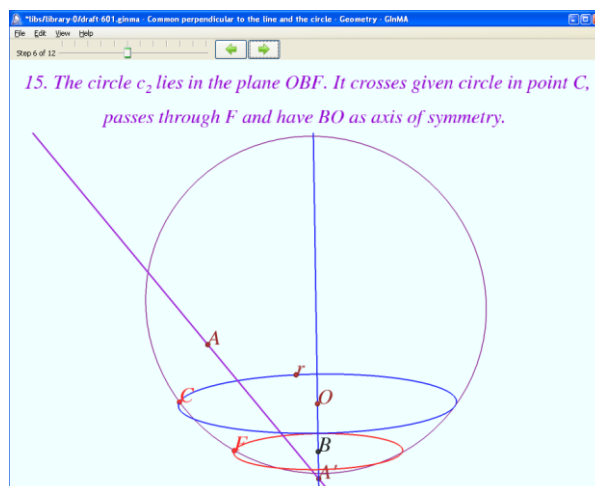


Figure 4.3.6 Step 16. The circle c_2 crosses BO at points S_1 and S_2 .

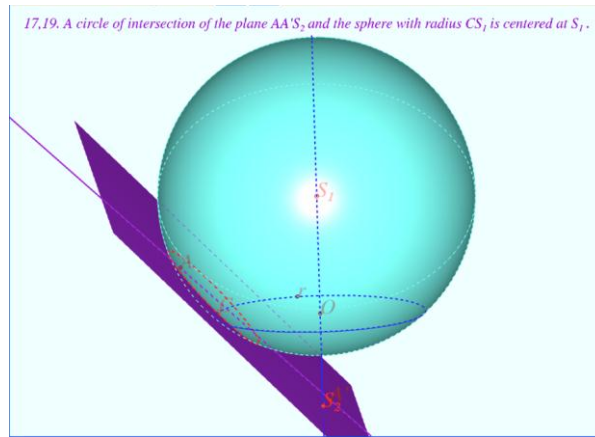


Figure 4.3.7 Steps 17, 19. A circle of intersection of the plane $AA'S_2$ and the sphere with radius CS_1 is centered at S_1 .

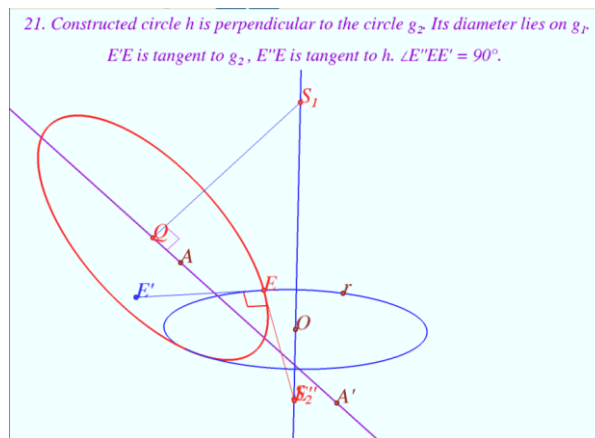


Figure 4.3.8 Step 21. Constructed circle h is perpendicular to the circle g_2 . Its diameter lies on g_1 . $E'E$ is tangent to g_2 , $E''E$ is tangent to h . $\angle E''EE' = 90^\circ$.

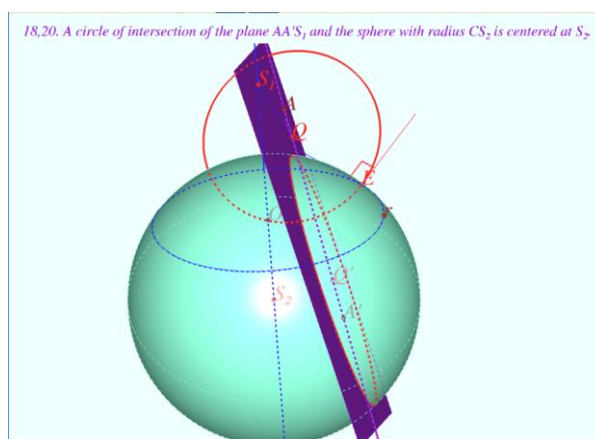


Figure 4.3.9 Steps 18, 20. A circle of intersection of the plane $AA'S_1$ and the sphere with radius CS_2 is centered at S_2 .

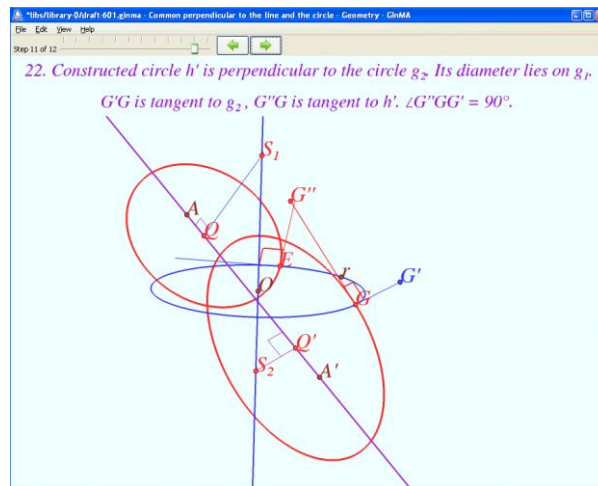


Figure 4.3.10 Step 22. Constructed circle h' is perpendicular to the circle g_2 . Its diameter lies on g_1 . $G'G$ is tangent to g_2 , $G''G$ is tangent to h' . $\angle G''GG' = 90^\circ$.

There is any difference between this solution and original one [Maeda, 2011]. To explain it we need a lemma .

Lemma. Let O be A projection onto the plane BCO , DD' is a common perpendicular to lines AB and CO . The angle between AB and CO is equal φ , $DE = DD'/\sin\varphi$. Prove that $\angle AEB = 90^\circ$.

Proof. Consider the view along CO . D' and C coincide with O , $AD' \perp BD'$, $AB \perp DD'$. Hence, the product of AD and BD projections is equal DD'^2 . Segments projections are $\sin\varphi$ times shorter than segments themselves. Hence $AD \cdot BD = DE^2$, therefore the angle AEB is right.

GInMA illustrations for lemma and its proof you can see in GInMA library as “Right angle construction” [24]. The screenshots are shown below, Figure 4.3.11.

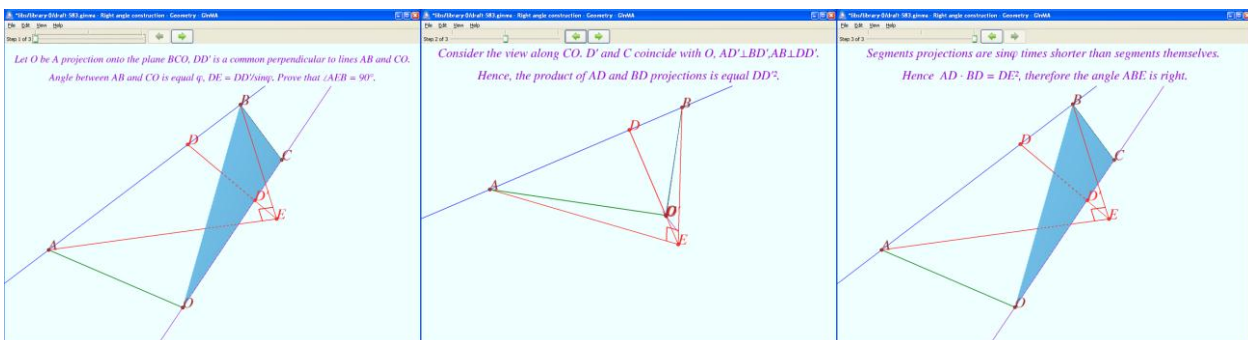


Figure 4.3.11 Right angle construction. (Plots produced with [GInMA], [24].)

We hope that these illustrations will make it easier to understand the problems invented by Yoichi Maeda and to see the beauty of solutions he had found. In our logical chains based on Yoichi Maeda ideas, amount of GInMA files three times greater than it is shown above. The problems discussed there are closely related with the problems described in this paper.

4. Conclusion

Dynamic geometry system GInMA allows to create and to explore the problems and the logic chains that use the geometric configurations. It develops students' creativity, thinking and calculation methods, and helps to understand multiple integrals. The established system is increasingly being used in Russian schools.

References

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Software Packages

[GInMA] GInMA, 2011, Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V. <http://deoma-cmd.ru/en/Products/Geometry/GInMA.aspx>

Supplemental Electronic Materials

All GInMA supplemental materials that accompany this paper can be found in GInMA library. Install the GInMA software from the website

<http://deoma-cmd.ru/en/Products/Geometry/GInMA.aspx>

Open GInMA library, choose “Papers” book in a books selection dialog.

Open “eJMT Development of creativity using 3D DGS GInMA” and open the GInMA files for the investigation of interactive solutions to problems shown in GInMA files [1] - [13].

- [1] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Curve divided by points.
- [2] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Plane divided by the circles.
- [3] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Division of space by intersecting spheres.
- [4] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Eight intersecting spheres with the centers in apexes of a cube.
- [5] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Volume of a spiral solid.
- [6] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Volume and surface area of a semicircle solid of revolution.
- [7] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Volume of a solid of revolution formed by rotating a cube around the diagonal.
- [8] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Solid common to two intersecting cylinders.
- [9] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Two perpendicular cylinders intersect a sphere.
- [10] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Shortest closed path inside a cube.
- [11] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Least volume of a solid common to two intersecting cubes.
- [12] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Three regular tetrahedra inside a cube.
- [13] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Circle tangent to all faces of quadrant.

For the investigation of interactive solutions to problems shown in GInMA files [14] – [23] in GInMA library choose “Papers” book, open “eJMT Yoichi Maeda papers illustrations ” and open GInMA files.

- [14] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Locus of points of the sphere
- [15] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Line in two-dimensional hyperbolic space H^2

[16] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Perpendicular to a line in the H^2 model

[17] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Perpendicular to line in the half-plane model

[18] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Perpendicular line in the half-space model

[19] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Perpendicular circles on the sphere

[20] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Common perpendicular in the half-space model (special case)

[21] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Common perpendicular in the half-space model

[22] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Common perpendicular in the H^3 Poincare model

[23] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Common perpendicular to the line and the circle

[24] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., GInMA file, Right angle construction

[25] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., video showing the solution to the problem of solid common to two intersecting cylinders using GInMA.

<http://www.youtube.com/watch?v=fwuXx1mJZw0>

[26] Nosulja, S., Shelomovskiy, D. and Shelomovskiy, V., video showing the solution to the problem of Common perpendicular to the geodesics in three-dimensional hyperbolic space H^3

<http://www.youtube.com/watch?v=kOnKuQyTpng>